

1	Kóp ózgeriwshili funkciyalar Kóp ózgeriwshili funkciya haqqında túsiniq. $\mathbf{R}^m$ keńisliktiń úles kóplikleri $\mathbf{m}$ ózgeriwshili funkciyanıń anıqlanıw oblastı sıpatında.
2	Eki ózgeriwshili funkciyanıń grafigi.
3	Qáddi sıızıqları hám betleri túsiniqi.
4	Kóp ózgeriwshili funkciyanıń limiti hám úzliksizligi $\mathbf{R}^m$ keńislikte noqattıń dógeregi. $\mathbf{R}^m$ keńisliktegi noqatlar izbe-izligi hám onıń limiti, $\mathbf{m}$ ózgeriwshili funkciyanıń limiti.
5	Tákirarıy limitler.
6	Tákirarıy limitler.
7	Úzliksizlik anıqlamaları. Kóp ózgeriwshili úzliksiz funkciyanıń qásiyetleri. Quramalı funkciyanıń úzliksizligi.
8	Kóp ózgeriwshili funkciyalardı differenciallaw. Dara tuwındılar.
9	Kóp ózgeriwshili differenciallanıwshı funkciya. Differenciallanıwshı bolıwınıń zárúrli, jetkilikli shártleri.
10	Kóp ózgeriwshili funkciyanıń tolıq differencialı. Differencialdıń juwıq esaplawğa qollanıwları. Bağıt boyınsha tuwındı. Gradient.
11	Eki ózgeriwshili funkciya ushın Teylor formulası. Ayqın emes funkciyanı differenciallaw.
12	Kóp ózgeriwshili funkciyalardıń ekstremumları Kóp ózgeriwshili funkciyanıń ekstremumları.
13	Ekstremumnıń zárúrli shárti. Eki ózgeriwshili funkciya ushın ekstremumnıń jetkilikli shárti.
14	Eń úlken hám eń kishi mánislerdi izlew.
15	Shártli ekstremumlar.

Tómendegi berilgen funkciyanıń anıqlanıw oblastın tabıń.

$$1. F(x, y) = \frac{1}{x + y}$$

$$2. F(x, y) = \sqrt{x} + \sqrt{y}$$

$$3. F(x, y) = \sqrt{1 - x^2 - y^2}$$

$$4. F(x, y) = \sqrt{y \sin x}$$

$$5. F(x, y) = \ln(x + y)$$

6. $F(x, y) = 1 + \sqrt{-(x - y)^2}$
7. $F(x, y) = \frac{1}{x-1} + \frac{1}{y}$
8. $F(x, y) = \sqrt{x^2 - 4} + \sqrt{1 - y^2}$
9. $F(x, y) = \sqrt{\sin(x^2 + y^2)}$
10. $F(x, y) = xy + \sqrt{\ln \frac{9}{x^2 + y^2}} + \sqrt{x^2 + y^2 - 9}$
11. $F(x, y) = \sqrt{(x^2 + y^2 - 1)(4 - x^2 - y^2)}$
12. $F(x, y) = \sqrt{x + y}$
13. $F(x, y) = \sqrt{-x} + \sqrt{y}$
14. $F(x, y) = \arccos \frac{x^2 + y^2}{9}$
15. $F(x, y) = \sqrt{x - \sqrt{y}}$
16. $F(x, y) = \operatorname{arctg} \frac{x - y}{1 + x^2 y^2}$
17. $F(x, y) = \ln\left(\frac{x^2}{9} - \frac{y^2}{4} - 1\right)$
18. $f(x, y) = \frac{\sqrt{4x - y^2}}{\ln(1 - x^2 - y^2)}$
19. $f(x, y) = \sqrt{\frac{x^2 + y^2 - x}{2x - x^2 - y^2}}$
20. $f(x, y) = \ln(-x - y)$
21. $f(x, y) = \arcsin\left(\frac{x}{y^2}\right) + \arcsin(1 - y)$
22. $\dot{f}(x, y) = \sqrt{\frac{x^2 + 2x + y^2}{x^2 - 2x + y^2}}$

Esseli limitlerdi esaplañ

$$1. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{a - \sqrt{a^2 - xy}}{xy}, \quad (a \neq 0)$$

$$2. \lim_{\substack{x \rightarrow 3 \\ y \rightarrow 0}} \frac{\tan(xy)}{xy}$$

$$3. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (x^2 + y^2) \sin\left(\frac{1}{xy}\right)$$

$$4. \lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \frac{x+y}{x^2+y^2}$$

$$5. \lim_{\substack{x \rightarrow \infty \\ y \rightarrow a}} \left(1 + \frac{y}{x}\right)^x$$

$$6. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{\sqrt{1+x^2y^2}-1}{x^2+y^2}$$

$$7. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{\sin(x^4y^2)}{(x^2+y^2)^2}$$

$$8. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (xy)^{2(x^2+y^2)}$$

$$9. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{e^{\frac{1}{x^4+y^4}}}{x^4+y^4}$$

$$10. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (1+x^2y^2)^{\frac{1}{x^2+y^2}}$$

$$11. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (1+x^2+y^2)^{\frac{1}{x^2+y^2}}$$

$$12. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{\sqrt[3]{x^4y^2}}{x^2+y^2}$$

$$13. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (x^2+y^2) \sin\left(\frac{1}{x^2+y^2}\right)$$

$$14. \lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} (x^2+y^2) \ln\left(1 + \sin \frac{1}{x^2+y^2}\right)$$

$$15. \lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \frac{x^2+y^2}{x^4+y^4}$$

$$16. \lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} (x+y)e^{-(x^2+y^2)}$$

$$17. \lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \frac{x^2 + y^2}{|x^3| + |y|^3}$$

$$18. \lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} (x^2 + y^2)^{|x|}$$

$$19. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (x^2 + y^2)x^2y^2$$

$$20. \lim_{\substack{x \rightarrow 1 \\ y \rightarrow 0}} \frac{\ln(x + e^y)}{\sqrt{x^2 + y^2}}$$

Tómendegi funkciyalardı úzliksizlikke tekseriń hám úzilis noqatın tabıń

$$1. f(x, y) = \frac{10x}{(x-1)^2 + (y-1)^2}$$

$$2. f(x, y) = \frac{3y}{2x - y}$$

$$3. f(x, y) = \begin{cases} \sqrt{1 - x^2 - y^2}, & \text{eger } x^2 + y^2 \leq 1 \\ 0, & \text{eger } x^2 + y^2 > 1 \end{cases} \quad \begin{matrix} \text{bolsa,} \\ \text{bolsa.} \end{matrix}$$

$$4. f(x, y) = \frac{x - y}{x + y}$$

$$5. f(x, y) = \frac{2x - 3}{x^2 + y^2 - 4}$$

$$6. f(x, y) = \frac{x - y^2}{x + y^2}$$

$$7. f(x, y) = \frac{x - y}{x^3 - y^3}$$

$$8. f(x, y) = \ln(9 - x^2 - y^2)$$

$$9. f(x, y) = \frac{3}{x^2 + y^2}$$

$$10. f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$$

$$11. f(x, y) = \frac{-xy}{x + y}$$

$$12. f(x, y) = \sin\left(\frac{1}{xy}\right)$$

$$13. f(x, y) = \frac{1}{\sin x \cdot \sin y}$$

$$14. f(x, y) = \cos\left(\frac{1}{x^2 + y^2 - 9}\right)$$

Takrariy limitti esaplań

$$1. \quad f(x, y) = \begin{cases} \frac{x - y + x^2 + y^2}{x + y}, & x \neq -y, \\ 0, & x = -y, \end{cases} \quad (0, 0)$$

$$2. \quad f(x, y) = \frac{x^2 + xy + y^2}{x^2 - xy + y^2}, \quad (0, 0)$$

$$3. \quad f(x, y) = \frac{\sin(x + y)}{2x + 3y}, \quad (0, 0)$$

$$4. \quad f(x, y) = \frac{\cos x - \cos y}{x^2 + y^2}, \quad (0, 0)$$

$$5. \quad f(x, y) = \frac{\sin |x| - \sin |y|}{\sqrt{x^2 + y^2}}, \quad (0, 0)$$

$$6. \quad f(x, y) = \frac{\sin 3x - \operatorname{tg} 2y}{6x + 3y}, \quad (0, 0)$$

$$7. \quad f(x, y) = \frac{x^2 + y^2}{x^2 + y^4}, \quad (\infty, \infty)$$

$$8. \quad f(x, y) = \frac{x^y}{1 + x^y}, \quad (\infty, 0)$$

$$9. \quad f(x, y) = \sin\left(\frac{nx}{2x + 3y}\right), \quad (\infty, \infty)$$

$$10. \quad f(x, y) = \frac{\sin x + \sin y}{x + y}, \quad (0, 0)$$

$$11. \quad f(x, y) = \frac{1}{xy} \cdot \operatorname{tg}\left(\frac{xy}{1 + xy}\right), \quad (0, \infty)$$

$$12. \quad f(x, y) = \log_x(x + y), \quad (1, 0)$$

$$13. \quad f(x, y) = \frac{x^2 \sin\left(\frac{1}{x}\right) + y}{x + y}, \quad (0, 0)$$

$$14. \quad f(x, y) = \begin{cases} \frac{x^2 - y^2}{|x| - |y|}, & |x| \neq |y|, \\ 0, & |x| = |y|, \end{cases} \quad (0, 0)$$

$\mathbb{R}^n$ кеңіслікте $\{x^{(n)}\}$ збе-ізліктің limitін есеплаң

$$1. \quad \{x^{(n)}\} = \left(\left(\frac{n-1}{n} \right)^5, \frac{n^3 + 27}{n^4 - 15} \right)$$

$$2. \quad \{x^{(n)}\} = \left(\frac{2^{n+2} + 3^{n+3}}{2^n + 3^n}, \frac{5 \cdot 2^n - 3 \cdot 5^{n+1}}{100 \cdot 2^n + 2 \cdot 5^n} \right)$$

$$3. \quad \{x^{(n)}\} = (\sqrt[3]{8}, \sqrt{0.5})$$

4. $\{x^{(n)}\} = \left(\frac{\sqrt[3]{8} - 1}{\sqrt[3]{2} - 1}, \frac{a+2}{4^{n+1}} \right)$
5. $\{x^{(n)}\} = \left((1+11^n)^{\frac{1}{n+2}}, a^{\frac{1}{n+p}} \right), a > 0, p > 0$
6. $\{x^{(n)}\} = \left(\sqrt[n]{n^2}, \sqrt[n]{n} \right)$
7. $\{x^{(n)}\} = \left(\sqrt[n]{3n-2}, \sqrt[n]{\frac{n^3+3n}{n^4-15}} \right)$
8. $\{x^{(n)}\} = \left(\frac{\log_3(n^2+1)}{n}, \frac{n - \lg n}{\log_2(4^n+1)} \right)$
9. $\{x^{(n)}\} = \left(\frac{(-2)^n}{(n+2)!}, \frac{1}{(0.3)^n n!} \right)$
10. $\{x^{(n)}\} = \left(\frac{n \cdot 3^n + 1}{n! + 1}, \frac{4^n + n^2 \cdot 2^n - 1}{n^4 + (n!)^2} \right)$
11. $\{x^{(n)}\} = \left(\sum_{k=1}^n \frac{1}{k(k+1)}, \prod_{k=1}^n 2^{1/2^k} \right)$

Екинши т ртипли туwунд сын таб н

1. $f(x, y) = xy - \frac{x}{y}$
2. $f(x, y) = (x^2 + y^2)^3$
3. $f(x, y) = x - 3 \sin y$
4. $f(x, y) = \frac{y}{x} e^{xy}$
5. $f(x, y) = \arctg(xy)$
6. $f(x, y) = y \sqrt[3]{x}$
7. $f(x, y) = \sqrt{2xy + y^2}$
8. $f(x, y) = \sin(xy)$
9. $f(x, y) = (1+x)^n (1+y)^n$
10. $f(x, y) = 2 \cos^2 \left(y - \frac{x}{2} \right)$
11. $f(x, y) = e^x \ln y + \sin y \cdot \ln x$
12. $f(x, y) = \operatorname{arccot}(x+2y)$

Ко рсетилген т ртипдеги туwунд сын  таб н

1. $f(x, y) = y \ln(xy), \quad \frac{\partial^3 f}{\partial x \partial y^2}$
2. $f(x, y) = \arctan\left(\frac{y}{x}\right), \quad \frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial y^2}$
3. $f(x, y) = x \cos y + y \sin x, \quad \frac{\partial^3 f}{\partial x^3}, \quad \frac{\partial^3 f}{\partial x^2 \partial y}, \quad \frac{\partial^3 f}{\partial y^3}$
4. $f(x, y) = \frac{x+y}{x-y}, \quad \frac{\partial^{10} f}{\partial x^6 \partial y^4}$
5. $f(x, y) = (x^2 + y^2)e^{x+y}, \quad \frac{\partial^{m+n} f}{\partial x^m \partial y^n}$
6. $f(x, y) = e^x \sin y, \quad \frac{\partial^{m+n} f}{\partial x^m \partial y^n}$

Taylor qatarına jayın

1. $f(x, y) = -x^2 + 2xy + 3y^2 - 6x - 2y - 4$ funksiyanın $n = 3$ bolǵandaǵı $(-2; 1)$ noqat dógeresindegi Taylor formulasın jazın
2. $f(x, y) = \sqrt{1 - x^2 - y^2}$ funksiyanın $n = 3$ bolǵandaǵı $(0; 0)$ noqat dógeresindegi Taylor formulasın jazın
3. $f(x, y) = e^x \sin y$ funksiyanın $n = 3$ bolǵandaǵı $(0; 0)$ noqat dógeresindegi Taylor formulasın jazın
4. $f(x, y) = \cos x \cdot \cos y$ funksiyanın $n = 3$ bolǵandaǵı $(0; 0)$ noqat dógeresindegi Taylor formulasın jazın
5. $f(x, y) = y^x$ funksiyanın $n = 2$ bolǵandaǵı $(1; 1)$ noqat dógeresindegi Taylor formulasın jazın